Also solved by P. Bracken, P. P. Dályay (Hungary), P. J. Fitzsimmons, E. J. Ionaşcu, O. Kouba (Syria), J. H. Lindsey II, O. P. Lossers (Netherlands), M. Omarjee (France), T. Persson & M. P. Sundqvist (Sweden), I. Pinelis, J. C. Smith, A. Stenger, R. Stong, and the proposer.

## Sum of Medians of a Triangle

**11790** [2014, 648]. *Proposed by Arkady Alt, San Jose, CA, and Konstantin Knop, St. Petersburg, Russia.* Given a triangle with semiperimeter $s$, inradius $r$, and medians of length $m_a$, $m_b$, and $m_c$, prove that $m_a + m_b + m_c \le 2s - 3(2\sqrt{3} - 3)r$.

*Solution by James Christopher Smith, Knoxville, TN.* Write $R$ for the circumradius. We use two inequalities. The first is

$$(m_a + m_b + m_c)^2 \le 4s^2 - 16Rr + 5r^2,$$

due to Xiao-Guang Chu and Xue-Zhi Yang. (See J. Liu, "On an inequality for the medians of a triangle," *Journal of Science and Arts,* **19** (2012) 127–136.) The second is

$$s \le (3\sqrt{3} - 4)r + 2R,$$

known as Blundon's inequality. (See problem E1935, this MONTHLY, **73** (1966) 1122.)

Write $u = 2\sqrt{3} - 3$. From Blundon's inequality,

$$\begin{aligned}(2s - 3ur)^2 &= 4s^2 - 12sur + 9u^2r^2 \\ &\ge 4s^2 - 12ur\big((3\sqrt{3} - 4)r + 2R\big) + 9u^2r^2 \\ &= 4s^2 - 24uRr + \big(9u^2 - 12u(3\sqrt{3} - 4)\big)r^2 \\ &= 4s^2 - 16Rr + (16 - 24u)Rr + 3u(7 - 6\sqrt{3}\,)r^2.\end{aligned}$$

Next, we use Euler's inequality $R \ge 2r$ to get

$$\begin{aligned}\big(2s - 3ur\big)^2 &\ge 4s^2 - 16Rr + (16 - 24u)2r^2 + 3u(7 - 6\sqrt{3}\,)r^2 \\ &= 4s^2 - 16Rr + 5r^2,\end{aligned}$$

which is greater than or equal to $(m_a + m_b + m_c)^2$ by the Chu–Yang inequality.

Also solved by R. Boukharfane (Canada), O. Geupel (Germany), O. Kouba (Syria), R. Tauraso (Italy), M. Vowe (Switzerland), and T. Zvonaru & N. Stanciu (Romania).

## A Middle Subspace

**11792** [2014, 648]. *Proposed by Stephen Scheinberg, Corona del Mar, CA.* Show that every infinite-dimensional Banach space contains a closed subspace of infinite dimension and infinite codimension.

*Solution by University of Louisiana at Lafayette Math Club, Lafayette, LA.* Let $V$ be an infinite-dimensional normed vector space (we do not require completeness). We construct a sequence of linearly independent vectors $v_0, v_1, \dots$ in $V$ and a sequence of bounded linear functionals $\lambda_0, \lambda_1, \dots$ such that $\lambda_i(v_j) = \delta_{i,j}$ for all nonnegative integers $i$ and $j$. Choose a nonzero $v_0 \in V$. By the Hahn–Banach theorem, there is a bounded linear functional $\lambda_0$ on $V$ with $\lambda_0(v_0) = 1$. Suppose that nonzero vectors $v_0, \dots, v_k \in V$ and bounded linear functionals $\lambda_0, \dots, \lambda_k$ have been defined such that $\lambda_i(v_j) = \delta_{i,j}$ for $i, j \in \{1, \dots, k\}$. The vector subspace $\bigcap_{i=1}^{k} \ker \lambda_i$ has infinite dimension since it has finite codimension in $V$, which is infinite-dimensional. In particular, there exists nonzero $v_{k+1} \in \bigcap_{i=1}^{k} \ker \lambda_i$. The